Cboe® Volatility Index

This document details the calculation methodology of the titled index/benchmark. This document, in conjunction with the Cboe Index Rules and Governance document (available on Cboe's Governance website), provides a transparent and easily accessible view of the methodology used to calculate the Cboe® Volatility Index ("VIX® Index"), ticker symbol 'VIX'.

Description of the Market or Economic Reality Measure

Cboe, in its capacity as a reporting authority, calculates and disseminates the Cboe Volatility Index commonly known as the "VIX Index" (ticker: VIX). The VIX Index is a financial benchmark designed to be an up-to-the-minute market estimate of the expected volatility of the S&P 500® Index, and is calculated by using the midpoint of real-time S&P 500® Index (SPX) option bid/ask quotes. More specifically, the VIX Index is intended to provide an instantaneous measure of how much the market expects the S&P 500 Index will fluctuate in the 30 days from the time of each tick of the VIX Index.

Intraday VIX Index values are based on snapshots of SPX option bid/ask quotes every 15 seconds and are intended to provide an indication of the fair market price of expected volatility at particular points in time. As such, these VIX Index values are often referred to as "indicative" or "spot" values. Cboe currently calculates VIX Index spot values between 2:15 a.m. CT and 8:15 a.m. CT (Cboe GTH session), and between 8:30 a.m. CT and 3:15 p.m. CT (Cboe RTH session).

As described in greater detail below, Cboe applies a filtering algorithm to the calculation of spot VIX Index values in order to identify and suppress VIX Index values that, while reflecting SPX option quotes at a particular point in time, do not reflect the expected volatility of the S&P 500 Index.

The VIX Index does not use contributed input data, and all of the input data is readily available via public sources. The VIX Index is non-significant, as defined by EU Regulation 2016/1011 ("EU Benchmark Regulation" or "EU BMR").

Index Calculations

The following describes the methodology for calculating the VIX Index, including applicable formulas and input data.

The generalized formula used in the VIX Index calculation[1] is:

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[1] See *More Than You Ever Wanted to Know About Volatility Swaps,* by Kresimir Demeterfi, Emanuel Derman, Michael Kamal and Joseph Zou, Goldman Sachs Quantitative Strategies Research Notes, March 1999, publicly available at:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) \quad - \quad \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \qquad (1)$$

where:

- σ $VIX/100 \Rightarrow VIX = \sigma \times 100$

- T   Time to expiration

- F   Forward index level derived from index option prices

- K_0 First strike below the forward index level, F

- K_i Strike price of ith out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$.

- ΔK_i Interval between strike prices – half the difference between the strike on either side of K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

  (*Note*: ΔK for the lowest strike is the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

- R   Risk-free interest rates to expiration

- $Q(K_i)$ The average of the bid quote and ask quote for each option with strike K_i.

Time to Expiration for Constituent Options

The VIX Index measures the 30-day expected volatility of the S&P 500 Index. The components of the VIX Index are at- and out-of-the-money put and call options with more than 23 days and less than 37 days to a Friday SPX expiration date. These include AM-settled SPX options with "standard" 3rd Friday expiration dates and PM-settled "weekly" SPX options that expire every Friday, except the 3rd Friday of

http://www.emanuelderman.com/writing/entry/more-than-you-ever-wanted-to-know-about-volatility-swaps-the-journal-of-der.

each month[2]. Once each week, the SPX options used to calculate the VIX Index "roll" to new contract maturities. For example, on the day before VIX futures and VIX options expiration, the VIX Index is generally calculated using two SPX option expirations: (1) one expiring 24 days later (i.e., "near-term") and, (2) one expiring 31 days later (i.e., "next-term"). On the following day, the SPX options that expire in 30 calendar days become the "near-term" options and the SPX options that expire a week later are "rolled" in as the "next-term" options.

The VIX Index calculation measures time-to-expiration in calendar days and divides each day into minutes in order to replicate the precision that is commonly used by professional option and volatility traders. N represents time-to-expiration in minutes and T represents time-to-expiration in years. The time-to-expiration is given by the following expressions:

$$N = M_{Current\ day} + M_{Settlement\ day} + M_{Other\ days}$$

$$T = N\ /\ \text{Minutes in a year}$$

where:

- $M_{Current\ Day}$ minutes remaining until midnight of the current day

- $M_{Settlement\ day}$ minutes from midnight until 8:30 a.m. for "standard" AM-settled SPX expirations; or minutes from midnight until 3:00 p.m. for "weekly" PM-settled SPX expirations

- $M_{Other\ days}$ total minutes in the days between current day and expiration day

Risk-Free Interest Rates

The risk-free interest rates, R_1 and R_2, are yields based on U.S. Treasury yield curve rates (commonly referred to as "Constant Maturity Treasury" rates), to which a cubic spline is applied to derive yields on the expiration dates of relevant SPX options. As such, the VIX Index value calculation may use different risk-free interest rates for near- and next-term options.

Selecting the options to be used in the VIX Index calculation

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[2] If the expiration date for Friday-expiring S&P 500 Index options is moved due to an exchange holiday, the time to expiration for constituent options will be adjusted accordingly.

The selected options are out-of-the-money SPX calls and out-of-the-money SPX puts centered around an at-the-money strike price, K_0. K_0 is defined as the strike price that equals or is immediately below the forward index level, *F*, for the near- and next-term options:

$$F_j = \text{Strike Price}_j + e^{R_j T_j} \times (\text{Call Price}_j - \text{Put Price}_j)$$

where:

- F_j Forward SPX level (j=1 for near-term maturity, j=2 for next-term maturity)
- Strike Price$_j$ The strike price at which the absolute difference between the Call Price$_j$ and Put Price$_j$ is smallest.
- R_j Risk-Free Interest Rate for j^{th} maturity
- T_j Time to expiration for j^{th} maturity
- Call Price$_j$ Average of Call bid quote and Call ask quote for j^{th} maturity
- Put Price$_j$ Average of Put bid quote and Put ask quote for j^{th} maturity

The near-term and next-term SPX options used in each VIX Index value calculation are selected using the following steps:

Out-of-the-money put options with strike prices < K_0

Start with the put strike immediately lower than K_0 and move to successively lower strike prices. Exclude any put option that has a bid price equal to zero (i.e., no bid). As shown below, once two puts with consecutive strike prices are found to have zero bid prices, no puts with lower strikes are considered for inclusion. (Note that the 2350 and 2355 put options are not included despite having non-zero bid prices.)

Put Strike	Bid	Ask	Include?
2345	0	0.15	*Not considered following two zero bids*
2350	**0.05**	**0.15**	
2355	**0.05**	**0.35**	
2360	0	0.35	No
2365	0	0.35	No
2370	0.05	0.35	Yes

2375	0.1	0.15	Yes
2380	0.1	0.2	Yes
.	.	.	.

Out-of-the-money call options with strike prices > K₀

Start with the call strike immediately higher than K_0 and move to successively higher strike prices, excluding call options that have a bid price of zero. As with the puts, once two consecutive call options are found to have zero bid prices, no calls with higher strikes are considered. (Note that the 3225 call option is not included despite having a non-zero bid price.)

Call Strike	Bid	Ask	Include?
.	.	.	.
3095	0.05	0.35	Yes
3100	0.05	0.15	Yes
3120	0	0.15	No
3125	0.05	0.15	Yes
3150	0	0.10	No
3175	0	0.05	No
3200	0	0.05	*Not considered following two zero bids*
3225	*0.05*	*0.10*	
3250	0	0.05	
.	.	.	

At-the-money call and put options with strike price = K₀

Select both the put and call with strike price K_0. Notice that two options are selected at K_0, while a single option, either a put or a call, is used for every other strike price.

Determining prices for the selected options

The price of each put option with a strike price < K_0 and each call option with a strike price > K_0 is the average of that option's bid quote and ask quote. For the purpose of the VIX Index calculation, the bid/ask quotes of the put and call with strike price equal to K_0 are combined into a single price.

$$Q(K_{i,j}<K_{0,j}) = (\text{Put Bid}_{i,j} + \text{Put Ask}_{i,j}) / 2$$

$$Q(K_{i,j}>K_{0,j}) = (\text{Call Bid}_{i,j} + \text{Call Ask}_{i,j}) / 2$$

$$Q(K_{i,j}=K_{0,j}) = (\text{Put Bid}_{i,j} + \text{Put Ask}_{i,j} + \text{Call Bid}_{i,j} + \text{Call Ask}_{i,j}) / 4$$

Determining the contribution of both near-term and next-term options

Applying the generalized VIX Index formula, set forth earlier, to the near-term and next-term options with time-to-expiration of T_1 and T_2, respectively, yields:

$$\sigma^2{}_1 = \frac{2}{T_1} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_1 T_1} Q(K_i) - \frac{1}{T_1}\left[\frac{F_1}{K_0}-1\right]^2$$

$$\sigma^2{}_2 = \frac{2}{T_2} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_2 T_2} Q(K_i) - \frac{1}{T_2}\left[\frac{F_2}{K_0}-1\right]^2$$

Generally, ΔK_i is half the difference between the strike prices on either side of K_i. For example, the ΔK for a SPX put option with a strike price of 2400 that is bracketed by a SPX 2375 put and a SPX 2410 put is 17.5: $\Delta K_{2400\ Put} = (2410 – 2375)/2$.

The ΔK for the options with the highest and lowest strike prices with the same maturity is the difference between that option's strike price and the strike price of the adjacent option. For example, if the lowest strike price was 2370 and the adjacent strike price was 2375, then $\Delta K_{2370\ Put}$ would be 5 (i.e., 2375 – 2370).

Combining $\sigma^2{}_1$ and $\sigma^2{}_2$ into a 30-day weighted average and calculating the VIX Index value:

$$\textbf{VIX Index =} \ 100 \times \sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T2}-N_{30}}{N_{T2}-N_{T1}}\right] + T_2\sigma_2^2\left[\frac{N_{30}-N_{T1}}{N_{T2}-N_{T1}}\right]\right\} \times \frac{N_{365}}{N_{30}}}$$

N_{T1} = number of minutes to expiration of the near-term options

N_{T2} = number of minutes to expiration of the next-term options

N_{30} = number of minutes in 30 days (30 x 1,440 = 43,200)

N_{365} = number of minutes in a 365-day year (365 x 1,440 = 525,600)

VIX Index Filtering Algorithm

As described above, "spot" VIX Index values are based on the average of SPX option bid/ask quotes ("mid-quote" prices), and only options that have a non-zero bid price are included. The bid-ask spread is generally accepted as a current indication of market price, and the average of the bid and ask quotes can be thought of as an indication of "fair" value. Spot VIX Index values are calculated using mid-quote option prices that are assumed to reflect these option fair values.

From time to time, option price quotations widen due to changing market conditions, technology failures or other reasons. When this occurs, options that were previously included in a VIX Index value calculation might be excluded due to them now having a zero-bid price. In other instances, the mid-quote prices of one or more SPX options might materially change. This can result in a VIX Index value that, while accurately reflecting SPX option quotes at the time, does not reflect the expected volatility of the S&P 500 Index.

The VIX Index Filtering Algorithm operates as follows

1. The first VIX Index spot value calculated during the Cboe RTH session or the Cboe GTH session is deemed to be the "baseline" VIX Index spot value.

2. Any VIX Index spot value calculated after and within two (2) minutes of the baseline that is higher than the baseline value or lower than the baseline value by .49 volatility points or less becomes the new baseline value.[3]

3. If VIX Index spot values calculated after and within two (2) minutes of a baseline are lower than the baseline by 0.50 volatility points or more, then the baseline VIX Index spot value will be republished as the VIX Index spot value.

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[3] Threshold parameters used in the VIX Index Filtering algorithm are subject to change at any time in response to changing levels of volatility or other market conditions. Cboe will provide notice to market participants for all such parameter changes.

4. If the published VIX Index spot values remain the same for a period of two (2) minutes because the calculated values are 0.50 or more volatility points lower than the baseline, the first VIX Index spot value calculated after the two-minute period becomes the new baseline VIX Index spot value.

5. The filtering algorithm does not apply to the first VIX Index spot value calculated during the Cboe RTH session (approximately 8:30 a.m. CT) or the first VIX Index spot value calculated during the Cboe GTH session (approximately 2:15 a.m. CT). All other VIX Index spot values calculated during Cboe RTH and Cboe GTH are subject to the filtering process.

Related VIX Values

In addition to the VIX Index, Cboe publishes the Cboe VIX Indicative Bid Index ("VWB"), a VIX Index value based on SPX bid quotes, and the Cboe VIX Indicative Ask Index ("VWA"), a VIX Index value based on SPX option ask quotes. These values provide a market estimate of SPX option bid-ask "spreads" expressed in volatility terms. Cboe also publishes volatility information related to the near-term and next-term VIX Index "components", σ_1 and σ_2, the Cboe Near-Term VIX Index ("VIN") and Cboe Far-Term VIX Index ("VIF"), respectively, every 15 seconds during each Cboe trading day. Cboe publishes these related VIX values during regular trading hours ("RTH") only.

Historical VIX Index Prices

Cboe makes available on its website more than 25 years of historical VIX Index values. Price history for the original Cboe Volatility Index (VXO) based on S&P 100® Index ("OEX® Index") options is available from 1986 to the present. Historical prices for the VIX Index, VXO Index and Cboe's other volatility indexes may be found on the Cboe website at http://www.Cboe.com/micro/IndexSites.aspx under Cboe Volatility Indexes.

Calculation and Dissemination

Cboe compiles, calculates, maintains, and disseminates all VIX Index values. The VIX Index is calculated and disseminated every 15 seconds between 2:15 a.m. CT and 8:15 a.m. CT (Cboe GTH session), and between 8:30 a.m. CT and 3:15 p.m. CT (Cboe RTH session).

Judgement and Potential Limitations in Calculation

No expert judgement or discretion is used by Cboe in performing the calculation of the VIX Index. Potential limitations for this index (i.e., situations where the index may not reflect the above described market or economic reality) include:

- where underlying index input data is unavailable, the VIX Index value will not be able to be calculated, and
- where the underlying option contract data is not available, the VIX Index value will not be able to be calculated.

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