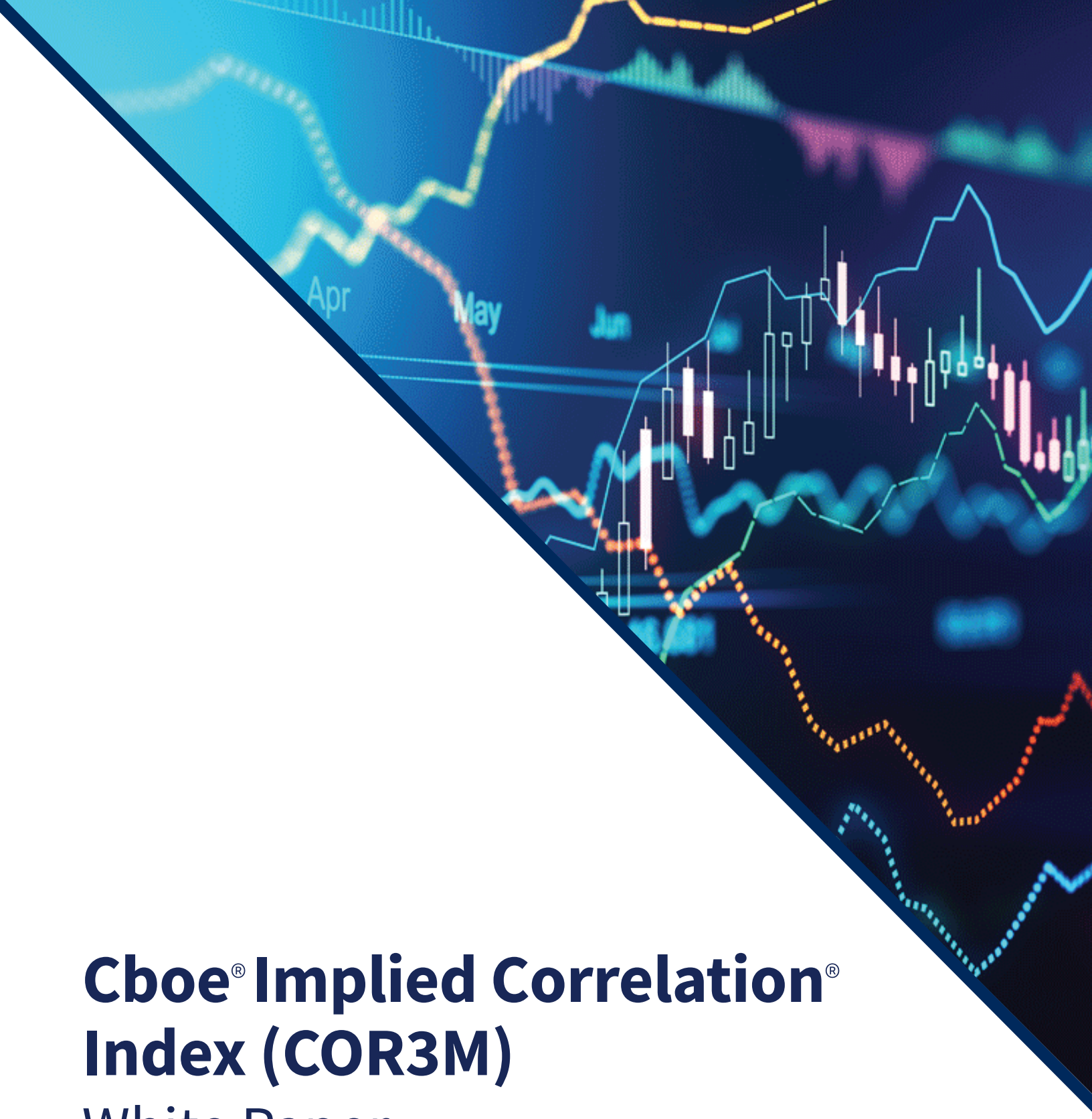

Cboe® Implied Correlation® Index (COR3M)

White Paper

Cboe®

Purpose

This document describes the methodology underlying the Cboe S&P 500 Implied Correlation Index. The index is structured to reflect changes in the relative premium between SPX index options and single-stock constituent options. Accounting for the implied volatility relationship between the SPX index option and a weighted basket portfolio of single-stock options, Cboe S&P 500 Implied Correlation Index measures the market's expectations of index component correlation. For a more rigorous overview of the index methodology, refer to the Cboe Implied Correlation Factsheet and Website.

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Introduction

Index volatility is driven by a combination of two factors: the individual volatilities of index components and the correlation of index component price returns. Intuitively, one would expect that the implied volatility of an index option would rise with a corresponding change in the implied volatilities of index component options. Yet, there are instances when the index option implied volatility moves without a corresponding shift in the implied volatilities of index component options.

The index measure isolates the impact of correlation changes on the index option implied volatility and provides a trading signal for volatility dispersion (correlation). Implied correlation provides a measure of the relative cheapness/richness of index options in relation to the index components. A long dispersion trade, which involves selling at-the-money index option straddles and purchasing at-the-money straddles in options on index components, would perform profitably under a high correlation regime.

Since the introduction of the Cboe Volatility Index (the VIX® index), Cboe has established a suite of tradable volatility products and indices that quantify forward-looking volatility expectations through theoretically robust metrics. Through the Cboe S&P 500® Implied Correlation Index, Cboe provides market participants with further granularity into the factors driving volatility, creates robust trading strategies and promotes greater transparency and integration of various products and strategies.

Summary Product Specifications

Name	Cboe Implied Correlation Index
Index Type	Equity Correlation
Volatility Unit	Delta Relative Calendar Day Price Volatility
Tenors	3M
Index Overview	The Cboe S&P 500 Implied Correlation Indices are the first widely disseminated, market-based estimate of the average correlation of the stocks that comprise the S&P 500® Index (SPX). The Cboe S&P 500 Implied Correlation Indexes offers insight into the relative cost of SPX options compared to the price of options on individual stocks that comprise the S&P 500.
Index Publisher	Cboe Global Markets
Methodology Outline	The SPX option implied variance can be defined using the Harry Markowitz's portfolio model[1], assuming the normality of returns, as follows: $$\sigma_{Index}^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j}$$ $$N = 50$$ The implied correlation is a measure of the average correlation between SPX index components, denoted as $\bar{\rho}$. The pairwise correlation, $\rho_{i,j}$, is replaced by the average correlation $\bar{\rho}$ in the expression above. The implied index value can be solved by isolating the average correlation term: The correlation index is calculated by first finding the difference between the SPX option implied variance and the implied variance of an uncorrelated portfolio of the top 50 SPX components by market capitalization. This value is then divided by the sum of pairwise weighted implied volatility products to calculate the implied correlation value.
Input Data	Feed of trading day ATM Implied Volatility of SPX index options and SPX Index components options for the maturities above from Hanweck.
Calculation Hours	8:30 a.m. to 3:00 p.m. Central Time (CT)

[1] Harry Markowitz, Portfolio Selection, The Journal of Finance, Vol. 7, No. 1. (Mar., 1952), pp. 77-91.

The Implied Correlation Index Calculation: Step-by-Step

The Implied Correlation Index is a financial benchmark that provide instantaneous market estimates of expected correlation using implied volatilities of SPX index and SPX component options. This index forms an essential tool to identify the drivers of Index Implied Volatility and evaluate the implications of major macroeconomic shocks on market expectations.

The implied correlation value does not rely on any options pricing model and uses ATM implied volatilities, defined by the strike at which the option delta equals 0.5.

This whitepaper shows a sample calculation of the Cboe 3M Implied Correlation Index. The implied correlation maturities can be calculated using the same methodology and differ only in the implied volatility tenors used. The generalized formula used is:

$$\rho_{Average} = \frac{\sigma_{Index}^2 - \sum_{i=1}^{N} w_i^2 \sigma_i^2}{2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j}$$

$$w_i = \frac{P_i * S_i}{\sum_{i=1}^{N} P_i * S_i}$$

Where

σ_{Index}	3 Month ATM Implied Volatility of SPX Index Option
σ_i	3 Month ATM Implied Volatility of i^{th} Index Component Option
P_i	Closing Price of i^{th} Index Component
S_i	Float Adjusted Shares Outstanding of i^{th} Index Component
N	Number of Stocks in Index Component Portfolio (50)

In the following sections, this paper goes over a sample implied correlation index calculation using sample data from February 10, 2021.

Step 1: Select the Relevant Index Components

For our backtested calculation at the end of each business day, SPX component weights are calculated using their float-adjusted market capitalizations. The top 50 components with the highest weights are included in the next business day's SPX tracking portfolio. Additionally, the 51st to 55th highest weighted components are included in the next business day's replacement pool. This pool is created to account for the event that one or more tracking portfolio components are excluded from the SPX index. In the event this occurs, non-zero weighted components of the replacement pool would replace excluded stocks in the tracking basket portfolio, forming a 50-stock SPX basket portfolio.

For the intraday calculation, we would similarly use the top 50 stocks as determined using the previous business day's closing adjusted market capitalizations. The SPX basket portfolio components and weights do not change intraday.

SPX Tracking Portfolio Selections					
AAPL	6.58%	**MA**	0.89%	**AVGO**	0.58%
MSFT	5.58%	**BAC**	0.76%	**MRK**	0.58%
AMZN	4.27%	**NFLX**	0.75%	**ABBV**	0.56%
FB	1.96%	**INTC**	0.73%	**NKE**	0.54%
TSLA	1.95%	**CMCSA**	0.73%	**CVX**	0.53%
GOOGL	1.89%	**ADBE**	0.72%	**QCOM**	0.50%
GOOG	1.83%	**VZ**	0.69%	**NEE**	0.50%
BRK/B	1.41%	**ABT**	0.67%	**ACN**	0.49%
JNJ	1.33%	**CRM**	0.66%	**MCD**	0.49%
JPM	1.29%	**XOM**	0.65%	**LLY**	0.49%
NVDA	1.07%	**CSCO**	0.62%	**TXN**	0.48%
V	1.06%	**T**	0.62%	**MDT**	0.48%
DIS	1.03%	**WMT**	0.61%	**COST**	0.48%
PYPL	1.01%	**PFE**	0.59%	**DHR**	0.45%
PG	0.97%	TMO	0.59%	**HON**	0.43%
UNH	0.95%	PEP	0.58%	**UNP**	0.42%
HD	0.90%	**KO**	0.58%		

SPX Replacement Pool Selections	
AMGN	0.42%
WFC	0.42%
BMY	0.41%
LIN	0.41%
PM	0.40%

All 50 tracking portfolio stocks will form the basket portfolio below, as none of the tracking portfolio stocks were excluded from the SPX index on February 10, 2021:

Basket Portfolio SPX Weights (February 10, 2021)					
AAPL	6.56%	MA	0.89%	AVGO	0.58%
MSFT	5.56%	BAC	0.76%	MRK	0.57%
AMZN	4.25%	NFLX	0.75%	ABBV	0.56%
FB	1.98%	INTC	0.73%	NKE	0.54%
TSLA	1.85%	CMCSA	0.73%	CVX	0.54%
GOOGL	1.90%	ADBE	0.72%	QCOM	0.50%
GOOG	1.84%	VZ	0.69%	NEE	0.50%
BRK/B	1.41%	ABT	0.67%	CAN	0.49%
JNJ	1.33%	CRM	0.66%	MCD	0.48%
JPM	1.29%	XOM	0.65%	LLY	0.49%
NVDA	1.11%	CSCO	0.60%	TXN	0.48%
V	1.06%	T	0.62%	MDT	0.48%
DIS	1.04%	WMT	0.61%	COST	0.48%
PYPL	1.01%	PFE	0.59%	DHR	0.45%
PG	0.96%	TMO	0.58%	HON	0.43%
UNH	0.96%	PEP	0.58%	UNP	0.42%
HD	0.91%	KO	0.58%	Sum Total 54.43%	

Step 2: Calculate Basket Portfolio Weights

Market capitalization weights of basket portfolio components are calculated using the following:

$$w_i = \frac{P_i * S_i}{\sum_{i=1}^{50} P_i * S_i} = \frac{W_i}{\sum_{i=1}^{50} W_i}$$

W_i	Weight of ith Basket Portfolio Stock in the SPX Index
w_i	Weight of ith Stock in the Basket Portfolio

On February 10th, AAPL's weight in the basket portfolio is **12.05%** (6.56%/54.43%). The weights of all stocks in the basket portfolio are shown below:

SPX Basket Portfolio Weights (February 10, 2021)					
AAPL	12.05%	**MA**	1.64%	**AVGO**	1.06%
MSFT	10.22%	**BAC**	1.40%	**MRK**	1.05%
AMZN	7.80%	**NFLX**	1.39%	**ABBV**	1.02%
FB	3.64%	**INTC**	1.34%	**NKE**	1.00%
TSLA	3.40%	**CMCSA**	1.34%	**CVX**	0.99%
GOOGL	3.49%	**ADBE**	1.32%	**QCOM**	0.92%
GOOG	3.39%	**VZ**	1.27%	**NEE**	0.92%
BRK/B	2.60%	**ABT**	1.24%	**CAN**	0.91%
JNJ	2.44%	**CRM**	1.21%	**MCD**	0.89%
JPM	2.37%	**XOM**	1.20%	**LLY**	0.90%
NVDA	2.04%	**CSCO**	1.11%	**TXN**	0.89%
V	1.95%	**T**	1.13%	**MDT**	0.88%
DIS	1.91%	**WMT**	1.11%	**COST**	0.87%
PYPL	1.85%	**PFE**	1.08%	**DHR**	0.84%
PG	1.77%	TMO	1.07%	**HON**	0.79%
UNH	1.76%	PEP	1.06%	**UNP**	0.76%
HD	1.67%	**KO**	1.07%	**Sum Total** 100.00%	

Step 3: Select Relevant Implied Volatilities

For each stock in the basket portfolio, we use Cboe Hanweck PhD 3M implied volatilities of 0.5 delta options. Implied volatilities quantify market expectations of 3M forward volatility. Implied volatilities are based on calendar days.

For more information on how the implied volatilities are calculated please refer to the Cboe Hanweck Methodology factsheet at **www.Cboe.com/Hanweck/OptionsAnalyticsContentOverview**.

SPX Basket Portfolio Implied Volatilities (February 10, 2021)					
AAPL	34.34%	**MA**	29.49%	**AVGO**	34.53%
MSFT	27.66%	**BAC**	33.63%	**MRK**	23.75%
AMZN	32.79%	**NFLX**	40.83%	**ABBV**	27.75%
FB	35.39%	**INTC**	33.37%	**NKE**	32.66%
TSLA	69.63%	**CMCSA**	26.24%	**CVX**	35.07%
GOOGL	30.26%	**ADBE**	32.38%	**QCOM**	38.73%
GOOG	29.99%	**VZ**	18.86%	**NEE**	28.13%
BRK/B	21.21%	**ABT**	26.48%	**CAN**	27.54%
JNJ	22.68%	**CRM**	35.95%	**MCD**	22.85%
JPM	29.40%	**XOM**	38.20%	**LLY**	31.99%
NVDA	47.99%	**CSCO**	26.40%	**TXN**	29.31%
V	26.98%	**T**	23.10%	**MDT**	27.40%
DIS	39.31%	**WMT**	27.27%	**COST**	24.55%
PYPL	41.04%	**PFE**	24.56%	**DHR**	27.18%
PG	20.44%	TMO	27.79%	**HON**	25.93%
UNH	30.21%	PEP	23.64%	**UNP**	27.95%
HD	26.50%	**KO**	23.07%	**SPX** 20.23%	

Step 4: Calculate Implied Correlation Index

Using the weight and implied volatility values for February 10, 2021, the implied correlation index can be calculated as follows:

Step A: Find Implied Variance of Index Option

$$\sigma_{Index}^2 = 0.2016^2 = \mathbf{0.04064}$$

Step B: Find Implied Variance of the Uncorrelated Basket Portfolio

$$\sum_{i=1}^{N} w_i^2\sigma_i^2 = (0.1205^2)(0.3434^2) + (0.1022^2)(0.2766^2) + \cdots + (0.0076^2) * \left(0.2795^2\right) = \mathbf{0.0049}$$

Step C: Find the Sum of Pairwise Weighted Implied Volatility Products

$$2\sum_{i=1}^{N-1}\sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j = 2[(0.1205 * 0.1022 * 0.3434 * 0.2766) + \cdots + (0.0079 * 0.0076 * 0.2593 * 0.2795)]$$
$$= \mathbf{0.0964}$$

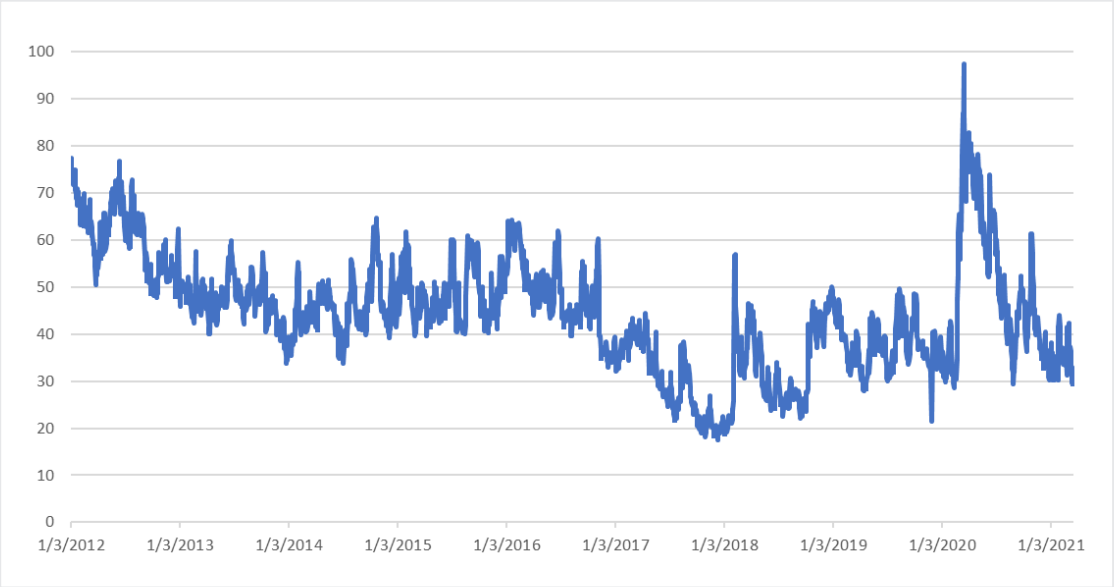
Step D: Calculate Implied Correlation:

$$\rho_{Average} = \frac{\sigma_{Index}^2 - \sum_{i=1}^{N} w_i^2\sigma_i^2}{2\sum_{i=1}^{N-1}\sum_{j>i}^{N} w_i w_j \sigma_i \sigma_j} = \frac{0.1110 - 0.0049}{0.0964} = \mathbf{0.3707}\ \blacksquare$$

Appendix: Theory and Implementation

Backtested Implied Correlation

Cboe 3-Month Implied Correlation Index:



Source: Cboe Hanweck and S&P SPICE (March 17, 2021)

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Historical Prices for the Cboe Implied Correlation Index may be found on the Cboe website at **Cboe.com/InsertLink/Here** under [**Insert**]. Contact us at:

John Hiatt
Derivatives Strategy
+1.312.786.7779
jhiatt@cboe.com

Parth Shah
Derivatives Strategy
+1.312.786.7157
pshah@cboe.com