Cboe Global Indices

Cboe Volatility Index

MATHEMATICS METHODOLOGY

Contents

1 Introduction

This document covers the mathematics of calculations for the VIX® Index and other Cboe volatility indices that use this methodology. It is intended to be read in conjunction with a family of White Papers that provide specific attributes for each Cboe volatility index, including the constituent options of the index, publication times, and other characteristics.

While there are several methods to create volatility indices, the methodology used to calculate the Cboe VIX Index and other Cboe volatility indices is based on theoretical work in pricing variance swaps to isolate exposure to volatility of an asset, independent of market conditions.[1] Cboe thanks Sandy Rattray, Devesh Shah, and Tim Klassen for their significant contributions to the development of the Cboe Volatility Index.

A key feature of Cboe volatility indices is that constituent options are weighted inversely proportional to the square of their strike strike (K^2). The weighting scheme used in the calculation of Cboe volatility indices matches the weighting scheme used to replicate variance swap payoffs with option portfolios. This, along with other elements of the methodology that seek to replicate volatility exposure using a portfolio of options, allows for the creation of volatility index derivatives with constant vega over a wide span of market movements.

2 Constituent Option Series Selection

Depending on the family of volatility indices, either the (a) Bracket Method with Constant Maturity Term or the (b) Nearest Term Method for Exclusion Criteria is used to select the “near-term” and “next-term” option series inputs for a Cboe volatility index given the specified target timeframe of expected volatility:

(a) Bracket Method

While each Cboe volatility index with “near-term” and “next-term” option series inputs seeks to measure a targeted time period of expected volatility, volatility indices that use the **Bracket Method** specify a “Constant Maturity Term,” (e.g., 30 days, 3 months, 6 months, etc.) as an element in the option series selection process. The length of the Constant Maturity Term for a particular Cboe volatility index is set forth in the relevant family of White Papers.

In addition to this Constant Maturity Term, the inputs for this method also include the set of option expirations that are candidates for near- and next-term expirations:

- The “near-term” options are defined to be the options within the provided set with days to expiration less than or equal to the Constant Maturity Term. If no options under this condition are found, then “near-term” options are defined to be options within the provided set expiring closest to the current date.

[1] *See Neuberger, 1996; Carr & Madan, 1998; Demeterfi, Derman, et al., 1999.*

(b) Nearest Term Method

The inputs for this method are the set of option expirations that are candidates for near- and next-term expirations as well as the exclusion criteria, which is a rule that determines which expiration dates should be excluded from this initial set.

The first step is to *exclude* from the provided set all option series where the exclusion criteria applies. For example, if options with a minimum of seven days to expiration are required as near-term option constituents, options that expire in fewer than seven days are excluded from the universe of candidate constituent options.

- The "near-term" options are defined to be the options within the remaining set expiring closest to the current date.

- The "next-term" options are defined to be the options within the remaining set expiring closest to and after the "near-term" options expiration date.

2 Interest Rate Calculation

2.1 Bounded Cubic Spline APY Rate

The risk-free interest rate, r_t, is calculated based on U.S. Treasury yield curve rates. The calculation process captures constant maturity Treasury (CMT) yields (i.e., bond equivalent yields) available on the U.S. Treasury website. Next a cubic spline is applied to interpolate/extrapolate a yield for each date between maturities, the bond equivalent yields (BEY) are converted to annualized percentage yields (APY), and then these yields are converted to continuously compounded interest rates for use in the Cboe volatility index calculation engine.

Bounded Cubic Spline Interpolation

The CMT yields (CMT_i) for the most recent business day are retrieved from the U.S. Treasury website. From this set, all null data points are excluded. A natural cubic spline method is applied to derive the bond equivalent yield (BEY) for any given time t. The corresponding number of days (t_i) used in the natural cubic spline interpolation for each fixed maturity found on the website are as follows:

Fixed maturity	1 Mo	2 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr
Number of days	30	60	91	182	365	730	1095	1825	2555	3650	7300	10950

The upper bound and lower bound for the BEY calculation is defined below:

- For interpolated periods $t_i < t < t_{i+1}$, where t_i and t_{i+1} are any two consecutive CMT maturities,
  - Lower bound is given by $\min(CMT_i,\ CMT_{i+1})$
  - Upper bound is given by $\max\ (CMT_i,\ CMT_{i+1})$

- For extrapolated periods $t < t_1$, where t_1 is the shortest available CMT maturity,
  - Lower bound b^{lower} is given by the equation $r = m_0^{\text{lower}} \times t + b^{\text{lower}}$.

Moreover,

$$m_0^{\text{lower}} = \frac{CMT_x - CMT_1}{t_x - t_1}; \qquad b^{\text{lower}} = CMT_1 - M_0^{\text{lower}} \times t_1$$

where

- (t_1, CMT_1) is the shortest available CMT maturity data point;
- (t_x, CMT_x) is the next shortest maturity data point such that $CMT_x \geq CMT_1$. If there is no such point (in the case of a complete inversion of the term structure – almost impossible), then let $m_0^{\text{lower}} = 0$.

- Upper bound b^{upper} is given by the equation $r = m_0^{\text{upper}} \times t + b^{\text{upper}}$.
  Moreover,

$$m_0^{\text{upper}} = \frac{CMT_z - CMT_1}{t_z - t_1}; \qquad b^{\text{upper}} = CMT_1 - M_0^{\text{upper}} \times t_1$$

where

- (t_1, CMT_1) is the shortest available CMT maturity data point;
- (t_z, CMT_z) is the next shortest maturity data point such that $CMT_z \leq CMT_1$. If there is no such point (in the case of no inversion in the term structure – a frequent occurrence), then let $m_0^{\text{upper}} = 0$.

Converting the BEY Rate to a Continuously Compounded APY Rate

Once BEY_t is calculated using the respective lower bound and upper bound, the risk-free interest rate r_t is calculated as follows:

$$APY_t = \left(1 + \frac{BEY_t}{2}\right)^2 - 1$$

$$r_t = \ln(1 + APY_t)$$

3 Volatility Index Calculations

(a) Single Term

The inputs for the single term volatility index calculation are the expiration date, interest rate, and the corresponding bid, ask, and option price for all selected options series. The generalized formula used in the volatility calculation is:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \qquad (1)$$

where

σ	**Volatility Index** $= \sigma \times 100$	K_0	First strike equal to or otherwise immediately below F

T	Time to expiration (in years)	K_i	Strike price of the i^{th} out-of-the-money (OTM) option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$
F	Option-implied forward price	ΔK_i	Interval between strike spreads: • Highest OTM Strike K_i: $K_i - K_{i-1}$ • Lowest OTM Strike K_i: $K_{i+1} - K_i$ • Otherwise: $(K_{i+1} - K_{i-1}) / 2$
R	Risk-free interest rate to expiration	$\mathrm{Q}(K_i)$	Option price of the OTM option with strike K_i; $\mathrm{Q}(K_0)$ is the average of the K_0 put option price and K_0 call option price

(i) Time to Expiration

The Cboe volatility index calculation measures time to expiration of a constituent option series, T, in calendar years. It is calculated by dividing the number of minutes until expiration of the selected options (rounded down to the nearest minute) by the number of minutes in a year. The time to expiration, T, is given by the following:

$$T = \left(M_{\text{Time to Expiry}}\right) / M_{365}$$

where

$M_{\text{Time to Expiry}}$	Number of minutes from time of calculation until expiration
M_{365}	Number of minutes in a 365-day year (365 x 1,440 = 525,600)

(ii) Forward Price and K_0

Determine the option-implied forward price level, F, by identifying the options strike price at which the absolute difference between the call price and the put price is smallest. If there are multiple put-call pairs with the same minimum absolute difference value, select the lowest strike price of these pairs. This strike is defined as the **at-the-money (ATM) strike**. In this subsection, the call and put prices reflect the midpoint of each candidate constituent option series' bid / ask quotes. Series with null quotes or bid price higher than ask price are not candidates to be the ATM strike.

Using the ATM strike call and put, the forward price, F, for the given term's constituent options is:

$$F = \text{Strike Price} + e^{RT} \times (\text{Call Price} - \text{Put Price})$$

Next, determine K_0, the strike price equal to or otherwise immediately below the forward price, F, for the near- and next-term candidate constituent options. If quotes of the K_0 put option or the K_0 call option are null or the bid price is higher than the ask price, then the Cboe volatility index cannot be calculated.

(iii) Strike Selection

First, remove all option strikes with null quotes from both the put and the call option series.

Then, select out-of-the-money put options with strike prices less than K_0. Start with the put option strike immediately lower than K_0 and move to successively lower strike prices. Exclude any put option that has a bid price or ask price equal to zero. Once two put options with consecutive strike prices are found to have zero bid prices or zero ask prices, exclude the observed put option(s) and consider no put options with lower strikes for inclusion. If all the out-of-the-money put options have been excluded, then the Cboe volatility index cannot be calculated.

Next, select out-of-the-money call options with strike prices greater than K_0. Start with the call option strike immediately higher than K_0 and move to successively higher strike prices. Exclude any call option that has a bid price or ask price equal to zero . As with the put options, once two call options with consecutive strike prices are found to have zero bid prices or zero ask prices, exclude the observed call option(s) and consider no call options with higher strikes for inclusion. If all the out-of-the-money call options have been excluded, then the Cboe volatility index cannot be calculated.

Finally, select **both** the put and call options with strike price K_0. Notice that two options are selected at K_0, while a single option, either a put or a call, is used for every other strike price.

The set of option series selected in this subsection comprise the constituent options for the volatility index calculation.

(iv) Calculating Volatility

The volatility index calculation combines the information reflected in the prices of all the selected constituent options. The contribution of a single option is proportional to ΔK and the price of that option, and inversely proportional to the square of the option's strike price.

Calculate the contribution of each strike by following these steps:

- Determine ΔK for each strike included in the calculation. Generally, ΔK_i is half the difference between the strike prices on either side of K_i. At the upper and lower edges of any given set of options, ΔK_i is simply the difference between K_i and the adjacent strike price.
- Compute the contribution by strike for each included option. For all puts $K_i < K_0$, the contribution is $e^{RT} \times \mathrm{Q}(K_i) \times (\Delta K_i / K_i^2)$. For all calls $K_i > K_0$, the contribution is $e^{RT} \times \mathrm{Q}(K_i) \times (\Delta K_i / K_i^2)$.
- For the strike K_0, the contribution is given by $e^{RT} \times \mathrm{Q}(K_0) \times (\Delta K_0 / K_0^2)$.

The contributions for each option strike are the summands in the sigma term of formula (1). Applying (1) to the given term's options with a time to expiration of T yields the following variance term:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

The Cboe volatility index value for the single term is therefore given by:

$$\textbf{Volatility Index} = \sigma \times 100$$

(b) Constant Maturity Term

Given the constant maturity term, expiration dates, and the variance (σ^2) for both terms, the formula used in the interpolated Cboe volatility index calculation is

$$\textbf{Volatility Index} = 100 \times \sqrt{\left\{T_1\sigma_1^2\left[\frac{M_{T_2} - M_{\mathrm{CM}}}{M_{T_2} - M_{T_1}}\right] + T_2\sigma_2^2\left[\frac{M_{\mathrm{CM}} - M_{T_1}}{M_{T_2} - M_{T_1}}\right]\right\} \times \frac{M_{365}}{M_{\mathrm{CM}}}}$$

where

M_{T_1}	The number of minutes until expiration of the near-term options
M_{T_2}	The number of minutes until expiration of the next-term options
M_{CM}	The number of minutes in the given constant maturity term
M_{365}	The number of minutes in a 365-day year (365 x 1,440 = 525,600)
T_i	M_{T_i} / M_{365}
σ_i^2	Variance of the i[th] term

4 Volatility Index Filtering Algorithm

(a) Single Term

The inputs for the single term volatility index calculation are the expiration date, interest rate, and the corresponding bid, ask, and option price for all selected options series. The generalized formula used in the volatility calculation is:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \qquad (1)$$

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F	Option-implied forward price	ΔK_i	Interval between strike spreads: • Highest OTM Strike K_i: $K_i - K_{i-1}$ • Lowest OTM Strike K_i: $K_{i+1} - K_i$ • Otherwise: $(K_{i+1} - K_{i-1})$ / 2
R	Risk-free interest rate to expiration	$Q(K_i)$	Option price of the OTM option with strike K_i; $Q(K_0)$ is the average of the K_0 put option price and K_0 call option price

(i) Time to Expiration

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Using the ATM strike call and put, the forward price, F, for the given term's constituent options is:

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First, remove all option strikes with null quotes from both the put and the call option series.

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Finally, select **both** the put and call options with strike price K_0. Notice that two options are selected at K_0, while a single option, either a put or a call, is used for every other strike price.

The set of option series selected in this subsection comprise the constituent options for the volatility index calculation.

(iv) Calculating Volatility

The volatility index calculation combines the information reflected in the prices of all the selected constituent options. The contribution of a single option is proportional to ΔK and the price of that option, and inversely proportional to the square of the option's strike price.

Calculate the contribution of each strike by following these steps:

- Determine ΔK for each strike included in the calculation. Generally, ΔK_i is half the difference between the strike prices on either side of K_i. At the upper and lower edges of any given set of options, ΔK_i is simply the difference between K_i and the adjacent strike price.
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The contributions for each option strike are the summands in the sigma term of formula (1). Applying (1) to the given term's options with a time to expiration of T yields the following variance term:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

The Cboe volatility index value for the single term is therefore given by:

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Given the constant maturity term, expiration dates, and the variance (σ^2) for both terms, the formula used in the interpolated Cboe volatility index calculation is

$$\textbf{Volatility Index} = 100 \times \sqrt{\left\{T_1 \sigma_1^2 \left[\frac{M_{T_2} - M_{\mathrm{CM}}}{M_{T_2} - M_{T_1}}\right] + T_2 \sigma_2^2 \left[\frac{M_{\mathrm{CM}} - M_{T_1}}{M_{T_2} - M_{T_1}}\right]\right\} \times \frac{M_{365}}{M_{\mathrm{CM}}}}$$

where

M_{T_1}	The number of minutes until expiration of the near-term options
M_{T_2}	The number of minutes until expiration of the next-term options
M_{CM}	The number of minutes in the given constant maturity term
M_{365}	The number of minutes in a 365-day year (365 x 1,440 = 525,600)
T_i	M_{T_i} / M_{365}
σ_i^2	Variance of the i[th] term

5 Calculation, Dissemination and Republication of Volatility Index Spot Values

(a) Index Level Filtering Algorithm

For any given trading session, the Filtering Algorithm, which is applied at the volatility index level, requires both a maximum republication interval, the "threshold period," and a threshold level (x volatility points).
The Index Filtering Algorithm operates as follows:

1. The first volatility index spot value calculated and disseminated during each trading session is deemed to be the "baseline" volatility index spot value.
2. Any volatility index spot value calculated after and within the threshold period (e.g., 5 minutes) of the baseline that is higher than the baseline value or lower than the baseline value by less than x volatility points becomes the new baseline volatility index spot value. That new baseline volatility index spot value will be disseminated.
3. If volatility index spot values calculated after and within the threshold period of a baseline are lower than the baseline volatility index spot value by x volatility points or more, then the baseline volatility index spot value will be republished as the volatility index spot value. Calculated volatility index spot values above the baseline are not filtered.
4. If the published volatility index spot values remain the same for the threshold period because the calculated values are x or more volatility points lower than the baseline, the first volatility index spot value calculated after the threshold period becomes the new baseline volatility index spot value. That new baseline volatility index spot value will be disseminated.
5. The Index Filtering Algorithm does not apply to the first volatility index spot value calculated and disseminated during the provided trading session. All other volatility index spot values calculated during the same trading session are subject to the filtering process. If the Filtering Algorithm is triggered, the calculated volatility index spot value will not be disseminated. Instead, the last published valid volatility index spot value will be republished.

(b) Volatility Index Spot Value Cannot be Calculated

1. As stated in the **Forward Price and K_0** section of this document, if quotes of the K_0 put option or the K_0 call option are null or the bid price is higher than the ask price, then the volatility index spot value cannot be calculated.
2. As stated in the **Strike Selection** section of this document, if all out-of-the-money call options have been excluded or all out-of-the-money put options have been excluded, then the volatility index spot value cannot be calculated.

In both cases where the volatility index spot value cannot be calculated, the last valid volatility index spot value is republished until a new valid volatility index spot value can be calculated and disseminated.

6 Series-Level Filtering Algorithm

Select Cboe indices utilize the series-level filtering algorithm. If the series-level filtering is applied to a given index, this will be specified in the relevant index methodology.
The series-level filtering algorithm is intended to be an overlay in the process of selecting the quotes that will be used as input for pre-specified index calculations with an objective of ensuring that the quote used in each constituent is reflective of the prevailing market in that series. Currently, the input quote in the calculation for a given timestamp t is the last quote available prior to t for each option series in the calculation. The algorithm below aims to add a filtering mechanism to this process to improve the quality of quotes used to calculate the indices.

This process is divided into three steps, which are detailed below.

Step 1. Retrieve Valid Quote Data

Given a constituent option series for an index calculation at timestamp t, determine both

i) Q_t^{last}, the last quote available in the session prior to timestamp t, and

ii) Q_t^{min}, the latest quote with minimum bid-ask spread within the 15-second time window prior to timestamp t

such that both Q_t^{last} and Q_t^{min} satisfy the following constraints:

- The bid and ask prices of the quote are numbers,
- The quote has a bid price greater than or equal to zero (i.e., bid ≥ 0), and
- The quote has an ask price strictly greater than the bid price (i.e., ask > bid).

If there is no valid quote prior to timestamp t that satisfies these constraints, then consider the selected quote to be null; i.e., take the bid price to be null and the ask price to be null. Note that while Q_t^{min} must be taken within 15 seconds prior to timestamp t, Q_t^{last} may be a persisted quote from any time prior to timestamp t in the session (as long as it is the last quote prior to t and fits the constraints above). Quote where bid prices and ask prices that match immediately prior quotes for a given option series are disregarded.

Step 2. Outlier Detection

For any given quote Q_t at time t, denote the following:

- The bid price of Q_t is $\text{Bid}(Q_t)$,
- The ask price of Q_t is $\text{Ask}(Q_t)$,
- The bid-ask spread of Q_t is $\text{Spread}(Q_t)$, where $\text{Spread}(Q_t) = \text{Ask}(Q_t) - \text{Bid}(Q_t)$,
- The midpoint price ("mid-price") of Q_t is $\text{Mid}(Q_t)$, where $\text{Mid}(Q_t) = \frac{\text{Bid}(Q_t) + \text{Ask}(Q_t)}{2}$.

If a given quote Q_t is null, then take $\text{Bid}(Q_t)$, $\text{Ask}(Q_t)$, $\text{Spread}(Q_t)$, and $\text{Mid}(Q_t)$ to all be null as well.

Calculate the Exponential Moving Average (EMA) of the Spreads

To define quotes as outliers, the series-level filtering algorithm first calculates the exponential moving average

(EMA) of the bid-ask spreads of a given constituent option series for time t using a recursive formula. At the initial time $t = 0$ of the session, the EMA is given by

$\mathrm{EMA}_0 = \mathrm{Spread}\left(Q_0^{\min}\right)$.

Note that EMA_0 may be null if $Q_0^{\min}$ is taken to be null; i.e., if there are no quotes available for the first calculation of the session.

For $t > 0$, define the EMA to be

$$\mathrm{EMA}_t = \begin{cases} \mathrm{EMA}_{t-1}, & \text{if } Q_t^{\min} \text{ is null} \\ \mathrm{Spread}\left(Q_t^{\min}\right), & \text{if } \mathrm{EMA}_{t-1} \text{ is null} \\ \alpha \times \mathrm{EMA}_{t-1} + (1-\alpha) \times \mathrm{Spread}\left(Q_t^{\min}\right), & \text{otherwise} \end{cases}$$

where α is a smoothing factor such that $0 \leq \alpha \leq 1$. Refer to the relevant index methodology for the value of α depending on the specific index.

Define and Flag Quotes as Outliers

Once the EMA of the spreads is calculated, proceed to find $\gamma(Q_t)$, a function that returns a constant equal to either γ_0, γ_1, or γ_2 (refer to the relevant index methodology for the numerical values of these constants γ_0, γ_1, and γ_2).

For any given quote Q_t at time t,

$$\gamma(Q_t) = \begin{cases} \gamma_0, & \text{if } \mathrm{Bid}(Q_t) = 0 \\ \gamma_1, & \text{if } \mathrm{Bid}(Q_t) > 0 \text{ and } \mathrm{Mid}(Q_t) \leq \mathrm{Mid}\left(\hat{Q}_{t-1}\right) \\ \gamma_2, & \text{if } \mathrm{Bid}(Q_t) > 0 \text{ and } \mathrm{Mid}(Q_t) > \mathrm{Mid}\left(\hat{Q}_{t-1}\right), \end{cases}$$

where $\hat{Q}_{t-1}$ denotes the filtered quote selected by the series-level filtering algorithm itself (after Step 3) for the previous calculation at time $t-1$ within the same session. In other words, the algorithm adopts a recursive approach that considers the last filtered quote when assessing how to filter the current quote.

If the given quote Q_t at time t meets any of the conditions below, then it is **not** defined as an outlier:

i) $\mathrm{Spread}(Q_t) \leq \gamma(Q_t) \times \mathrm{EMA}_t$,

ii) $\mathrm{Spread}(Q_t) \leq \lambda$,

iii) $\mathrm{Bid}(Q_t) > \mathrm{Mid}\left(\hat{Q}_{t-1}\right)$,

iv) $\mathrm{Ask}(Q_t) < \mathrm{Mid}\left(\hat{Q}_{t-1}\right)$ and $\mathrm{Bid}(Q_t) > 0$.

Refer to the relevant index methodology for the value of λ, the maximum spread parameter, depending on the specific index. Note that quotes Q_t are not flagged as outliers for the first calculation of a session or when EMA_{t-1} is null.

Step 3. Determine the Filtered Quotes

With the relevant quotes flagged as outliers, the final filtered quote $\hat{Q}_t$ at time t may be determined as follows:

- If the last valid quote Q_t^{last} is **not** an outlier, then simply take $\hat{Q}_t = Q_t^{\mathrm{last}}$.

- Otherwise, if the last valid quote Q_t^{last} is an outlier, then
    i. If Q_t^{min} is not a null quote and not an outlier, then take $\hat{Q}_t = Q_t^{\text{min}}$.
    ii. Otherwise, take $\hat{Q}_t = \hat{Q}_{t-1}$, the previous calculation's filtered quote.

This filtered quote $\hat{Q}_t$ of a given option series serves as input for the index calculation at timestamp t. Note that the steps above imply that if the last valid quote is an outlier, then the quote with the minimum bid-ask spread is used in the index calculation instead if it is not defined as an outlier. Moreover, if this quote with minimum bid-ask spread is also an outlier, then the previous calculation's filtered quote is persisted and used in the index calculation. Again, quotes are not filtered in the first calculation of the session.

Quote Filtering Example 1: $\hat{\boldsymbol{Q}}_{\boldsymbol{t}} = \boldsymbol{Q}_{\boldsymbol{t}}^{\boldsymbol{min}}$

As an example of the series-level filtering algorithm in practice, consider the VXMXEA Index, the Cboe MSCI EAFE Volatility Index, which takes underlying MXEA options as input. On the trade date May 3rd, 2023 for timestamp t = 15:19:30 (i.e., 3:19:30 p.m. ET), the MXEA option series of calls expiring May 19th, 2023 with strike price 2090 includes the quotes below in the 15-second time window prior to t.

MXEA 2090 Calls Expiring 5/19/2023 with timestamp t = 15:19:30				
Time of Quote	Bid	Ask	Spread	Mid
15:19:19.255645	54.8	58.9	4.1	56.85
15:19:19.255967	54.8	59.3	4.5	57.05
15:19:19.822725	54.4	58.9	4.5	56.65
15:19:20.138311	54.6	59.1	4.5	56.85
15:19:20.261043	54.6	59.1	4.5	56.85
15:19:21.588101	54.9	59.1	4.2	57
15:19:21.588945	54.9	59.4	4.5	57.15
15:19:25.951666	54.9	59.4	4.5	57.15
15:19:26.025636	54.9	59.3	4.4	57.1
15:19:26.029053	54.8	59.3	4.5	57.05
15:19:26.444674	54.9	59.3	4.4	57.1
15:19:26.445398	54.9	59.4	4.5	57.15
15:19:27.525609	54.9	59.4	4.5	57.15
15:19:27.527957	49.6	64.6	15	57.1
15:19:28.122755	50.1	65.1	15	57.6
15:19:28.690431	50.1	65.1	15	57.6
15:19:29.907117	50.3	65.1	14.8	57.7
15:19:29.908263	50.3	65.3	15	57.8

Following Step 1 of the algorithm, Q_t^{last} is the quote at 15:19:29.908263 with bid 50.3 and ask 65.3 since it is the last valid quote before the timestamp t = 15:19:30 (highlighted in red in the table above). For Q_t^{min}, take the quote at 15:19:19.255645 with bid 54.8 and ask 58.9 since it is the latest quote with the minimum bid-ask spread (4.1) in the 15-second time window before t (highlighted in green in the table above).

In this case, $t > 0$ so this is not the first calculation of the session. To apply Step 2 then, suppose running the algorithm on the same option series for the previous calculation at time $t - 1$ yields the following values:

Previous timestamp ($t-1$)	EMA_{t-1}	$\text{Bid}(\hat{Q}_{t-1})$	$\text{Ask}(\hat{Q}_{t-1})$	$\text{Mid}(\hat{Q}_{t-1})$
15:19:15	5.199	54.4	58.9	56.65

Since Q_t^{min} and EMA_{t-1} are not null, the EMA for time t is

$$\text{EMA}_t = \alpha \times \text{EMA}_{t-1} + (1-\alpha) \times \text{Spread}(Q_t^{\text{min}})$$
$$\text{EMA}_t = 0.95 \times 5.199 + (1-0.95) \times 4.1 = 5.14405$$

With the EMA of the spreads calculated, proceed to determine if Q_t^{last} is an outlier. The quote $Q_t = Q_t^{\text{last}}$ satisfies the conditions that

- $\text{Bid}(Q_t) > 0$ (i.e., $50.3 > 0$), and
- $\text{Mid}(Q_t) > \text{Mid}(\hat{Q}_{t-1})$ (i.e., $57.8 > 56.65$)

so $\gamma(Q_t) = \gamma_2 = 2.5$. (Here $\alpha = 0.95$, $\gamma_2 = 2.5$, and $\lambda = 0.5$ for VXMXEA as specified in the index methodology)
Moreover, for the quote $Q_t = Q_t^{\text{last}}$ observe that

i) $\text{Spread}(Q_t) \nleq \gamma(Q_t) \times \text{EMA}_t$ because $15 \nleq 2.5 \times 5.14405$,

ii) $\text{Spread}(Q_t) \nleq \lambda$ because $15 \nleq 0.5$,

iii) $\text{Bid}(Q_t) \ngtr \text{Mid}(\hat{Q}_{t-1})$ because $50.3 \ngtr 56.65$, and

iv) $\text{Bid}(Q_t) > 0$ but $\text{Ask}(Q_t) \nless \text{Mid}(\hat{Q}_{t-1})$ because $50.3 > 0$ but $65.3 \nless 56.65$.

Therefore $Q_t = Q_t^{\text{last}}$ meets none of the conditions listed in Step 2 and is defined as an outlier. Repeating this procedure to determine if Q_t^{min} is an outlier, the quote $Q_t = Q_t^{\text{min}}$ satisfies the conditions that

- $\text{Bid}(Q_t) > 0$ (i.e., $54.8 > 0$), and
- $\text{Mid}(Q_t) > \text{Mid}(\hat{Q}_{t-1})$ (i.e., $56.85 > 56.65$)

so $\gamma(Q_t) = \gamma_2 = 2.5$. Furthermore, the quote $Q_t = Q_t^{\text{min}}$ meets condition (i) in Step 2:

$\text{Spread}(Q_t) \leq \gamma(Q_t) \times \text{EMA}_t$ because $4.1 \leq 2.5 \times 5.14405$.

Therefore $Q_t = Q_t^{\text{min}}$ is not an outlier.

Proceeding with Step 3, the last valid quote Q_t^{last} is an outlier, but note that Q_t^{min} is not a null quote and Q_t^{min} is not an outlier. The filtered quote $\hat{Q}_t = Q_t^{\text{min}}$ then (i.e., the quote with the minimum bid-ask spread having bid 54.8 and ask 58.9 is used in the index calculation for this particular timestamp and option series instead of the last quote).

Quote Filtering Example 2: $\hat{Q}_t = \hat{Q}_{t-1}$

As another example of the filtering process, again consider the VXMXEA Index on the same trade date of May 3rd, 2023 for timestamp t = 15:28:00 (i.e., 3:28:00 p.m. ET). The MXEA option series of calls expiring May 19th, 2023 with strike price 2090 includes the quotes below in the 15-second time window prior to t.

MXEA 2090 Calls Expiring 5/19/2023 with timestamp t = 15:28:00				
Time of Quote	**Bid**	**Ask**	**Spread**	**Mid**
15:27:55.437276	50.3	65.1	14.8	57.7
15:27:55.437717	50.1	65.1	15	57.6

In this case, Q_t^{last} is the quote at 15:27:55.437717 with bid 50.1 and ask 65.1 since it is the last valid quote before the timestamp t = 15:28:00 (highlighted in red in the table above). For Q_t^{min}, take the quote at 15:27:55.437276 with bid 50.3 and ask 65.1 since it is the latest quote with the minimum bid-ask spread (14.8) in the 15-second time window before t (highlighted in green in the table above).

Since $t > 0$, this is not the first calculation of the session. Assume running the algorithm on the same option series for the previous calculation at time $t - 1$ yields the following values:

Previous timestamp ($t - 1$)	EMA_{t-1}	$\text{Bid}(\hat{Q}_{t-1})$	$\text{Ask}(\hat{Q}_{t-1})$	$\text{Mid}(\hat{Q}_{t-1})$
15:27:45	4.884	55.2	59.7	57.45

Since Q_t^{min} and EMA_{t-1} are not null, the EMA for time t is

$$\text{EMA}_t = \alpha \times \text{EMA}_{t-1} + (1 - \alpha) \times \text{Spread}(Q_t^{\text{min}})$$
$$\text{EMA}_t = 0.95 \times 4.884 + (1 - 0.95) \times 14.8 = 5.3798.$$

Now first determine if Q_t^{last} is an outlier. The quote $Q_t = Q_t^{\text{last}}$ satisfies the conditions that

- $\text{Bid}(Q_t) > 0$ (i.e., $50.1 > 0$), and
- $\text{Mid}(Q_t) > \text{Mid}(\hat{Q}_{t-1})$ (i.e., $57.6 > 57.45$)

so $\gamma(Q_t) = \gamma_2 = 2.5$. Moreover, for the quote $Q_t = Q_t^{\text{last}}$ observe that

v) $\text{Spread}(Q_t) \nleq \gamma(Q_t) \times \text{EMA}_t$ because $15 \nleq 2.5 \times 5.3798$,

vi) $\text{Spread}(Q_t) \nleq \lambda$ because $15 \nleq 0.5$,

vii) $\text{Bid}(Q_t) \ngtr \text{Mid}(\hat{Q}_{t-1})$ because $50.1 \ngtr 57.45$, and

viii) $\text{Bid}(Q_t) > 0$ but $\text{Ask}(Q_t) \nless \text{Mid}(\hat{Q}_{t-1})$ because $50.1 > 0$ but $65.1 \nless 57.45$.

Therefore $Q_t = Q_t^{\text{last}}$ meets none of the conditions listed in Step 2 and is defined as an outlier. Repeating this procedure to determine if Q_t^{min} is an outlier, the quote $Q_t = Q_t^{\text{min}}$ satisfies the conditions that

- $\text{Bid}(Q_t) > 0$ (i.e., $50.3 > 0$), and
- $\text{Mid}(Q_t) > \text{Mid}(\hat{Q}_{t-1})$ (i.e., $57.7 > 57.45$)

so $\gamma(Q_t) = \gamma_2 = 2.5$. For the quote $Q_t = Q_t^{\text{min}}$, checking the conditions in Step 2 yields

i) $\text{Spread}(Q_t) \nleq \gamma(Q_t) \times \text{EMA}_t$ because $14.8 \nleq 2.5 \times 5.3798$,

ii) $\text{Spread}(Q_t) \nleq \lambda$ because $14.8 \nleq 0.5$,

iii) $\text{Bid}(Q_t) \ngtr \text{Mid}(\hat{Q}_{t-1})$ because $50.3 \ngtr 57.45$, and

iv) $\text{Bid}(Q_t) > 0$ but $\text{Ask}(Q_t) \nless \text{Mid}(\hat{Q}_{t-1})$ because $50.3 > 0$ but $65.1 \nless 57.45$.

Since $Q_t = Q_t^{\text{min}}$ meets none of the conditions listed in Step 2, it is also an outlier.

Finally, using the rules in Step 3, the last valid quote Q_t^{last} is an outlier and Q_t^{min} is also an outlier. Thus, the filtered quote $\hat{Q}_t = \hat{Q}_{t-1}$ (i.e., the previous calculation's filtered quote with bid 55.2 and ask 59.7 is used in the index calculation for this particular timestamp and option series instead of the last quote).

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Appendix 1 - Changes

Major changes described in this document since February 10, 2025, are as follows:

Change Summary	Effective Date	Previous Language	Updated Language
Modifications to the Strike Selection in Volatility Index Calculations	February 10, 2025	Historically, put and call options that have a bid price equal to zero have been excluded.	Per the effective date, put and call options that have a bid price or ask price equal to zero will be excluded.
Inclusion of Series Level Filtering Algorithm section	August 25, 2025	The "Series-Level Filtering Algorithm" was previously included as an appendix in the Cboe MSCI Volatility Indices Methodology.	The "Series-Level Filtering Algorithm" has now been integrated as a formal section within this document.

Appendix 2 – Document Information

Version Number	5.0
Last Revised Date	February 26, 2026

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