Cboe American-Style Options Implied Volatility Calculation Methodology

This document details a methodology used to construct a volatility surface from American settlement style options. This volatility surface can be used to generate intra-day and end-of-day indicative prices for existing or hypothetical option series.

Implied Volatility Calculation Method

Discrete Dividends and Interest Rates

The discrete dividend is collected from Cboe/Hanweck. The Cboe/Hanweck applies a hybrid analyst+algorithm system for predicting future dividend payments and dates associated with an array of securities across the global market. The Cboe/Hanweck incorporates a database of corporate actions to build up dividend payments patterns, identifying changes in shares in issue (through rights issues, consolidations, bonus issues, and subdivisions), and recording various types of disbursements.

The algorithmic calculations process follows a series of rules and involves:

- Inspecting dividend history.
- Incorporating changes to the issued share capital.
- Identifying patterns.
- Calculating individual forward amounts and dates two fiscal years ahead.

Where a clear pattern cannot be identified (e.g., due to a recent change in the fiscal year-end date, a resumption or cut of dividend, or alteration in the dividend payment pattern); the security is flagged to the analysts, who perform individual market research to forecast the data.

The analysts maintain and monitor the history of the accuracy of forecasts vs. actual outcomes to control and improve their forecasts’ quality. For more information about Cboe/Hanweck, visit [https://www.hanweck.com/](https://www.hanweck.com/).

The risk-free interest rates are yields based on U.S. Treasury yield curve rates (commonly referred to as “Constant Maturity Treasury” rates), to which a cubic spline is applied to derive yields on the expiration dates.

The forward price is calculated as:

$$ F_T(\text{Divs to Expiry } T) = \sum_{1 \leq t \leq T} D_t e^{r_t(T-t)} $$

Where:
Calculation of Implied Volatility

The implied volatility \( V \) for each maturity and strike level is calculated by equating the Cox-Ross-Rubinstein Binomial Option Price to the observed option price from the market and the dividends/interest rates calculated in the section above. For each maturity,

\[ |\text{Model Price}_i - \text{Option Price}_i| < \text{Tolerance} \]

Smile Interpolation and Extrapolation

To calculate the implied volatility for a non-listed strike and a non-listed maturity, we utilize two different techniques for smile interpolation and extrapolation: (i) the spline interpolation for smile interpolation and (ii) the classical SABR model for smile extrapolation.

Spline Interpolation

The spline interpolation is used for the strike range \( K_- \leq K \leq K_+ \), and the expiration range \( T_- \leq T \leq T_+ \). We use a cubic spline interpolation in total implied variance space. This is a method to interpolate unknown values between given points using piecewise third order polynomials. The idea is to fit a piecewise function of the form:

\[
f(x) = \begin{cases} 
  f_1(x) & x_1 \leq x < x_2 \\
  f_2(x) & x_2 \leq x < x_3 \\
  \vdots \\
  f_{n-1}(x) & x_{n-1} \leq x < x_n 
\end{cases}
\]

\[ x = K \]
\[ f(x) = V(K) \]

For each \( f_i \) is a third-order polynomial defined by:

\[
f_i(x) = \alpha_i x^3 + \beta_i x^2 + \gamma_i x + \delta_i \\
f_i(x_i) = \alpha_i x_i^3 + \beta_i x_i^2 + \gamma_i x_i + \delta_i = f_{i-1}'(x_i)
\]

Next, we make the first and second derivatives agree at each adjacent points.

\[
f_i'(x) = 3\alpha_i (x - x_i)^2 + 2\beta_i (x - x_i) + \gamma_i = f_{i-1}''(x) \\
f_i''(x) = 6\alpha_i (x - x_i) + 2\beta_i = f_{i-1}'''(x)
\]
Now, we have 4(n-1) coefficients and 4(n-1) equations, therefore, from basic algebra, we could derive the coefficients, and the volatility between known implied volatility.

**SABR Model**

The SABR model extrapolates the smile for $0 \leq K(T) \leq K(T)$ — (left extrapolation) and $K(T) + \leq K(T)$ (right extrapolation).

The SABR model was introduced as a simple class of stochastic volatility processes for the underlying. Given traded and liquid options, we fit the SABR model on the observed smile and estimate the parameters. Using these parameters, we can estimate implied volatility to price at various points on the volatility surface.

The SABR model assumes that the forward rate and the instantaneous volatility are driven by two correlated Brownian motions:

$$
\begin{align*}
    df_t &= \alpha_t F_t^\beta dW^1_t \\
    d\alpha_t &= \nu \alpha_t dW^2_t \\
    E[dW^1_t dW^2_t] &= \rho dt
\end{align*}
$$

where:

- $\alpha$ is the instantaneous vol;
- $\nu$ is the vol of vol;
- $\rho$ is the correlation between the Brownian motions driving the $F$ and the $\nu$;
- $\beta$ is the leverage effect;

The formula that the implied volatility must satisfy is:

$$
|SABR \text{ Implied Vol} - \text{Observed Implied Vol}| < \text{Tolerance}
$$

$$
SABR \text{ Implied Vol}(T, F, K, \alpha, \beta, \nu, \rho) = \frac{az}{\chi(z)FK^{(1-\beta)/2}} \left( 1 + \frac{(1-\beta)^2(LnF/K)^2}{24} + \frac{(1-\beta)^4(LnF/K)^4}{1920} \right)^{-1} \times \left( 1 + \frac{(1-\beta)^2(\alpha)^2}{24(FK)^{1-\beta}} + \frac{\alpha\beta\nu\rho}{4(FK)^{1-\beta/2}} + \frac{(2-3\rho^2)\nu^2}{24} \right) T,
$$

Where:

$$
z = \frac{\nu \ln F}{K} F K^{\frac{1-\beta}{T}}, \chi(z) = \ln \left( \frac{1-2\rho z + z^2 + z - \rho}{1-\rho} \right)
$$

Given the initial value of the forward, $F$, the model is governed by 4 parameters, $\alpha, \beta, \nu, \rho$. 
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